COMPOUND ANGLE FORMULAE

1. Solve each of the following equations for $0^\circ \le x^\circ \le 360^\circ$

(a) $\cos 2x^\circ - 3\sin x^\circ = 2\sin^2 x^\circ$ (b) $3\cos 2x^\circ - 2\sin x^\circ - 1 = 0$

2. Solve each of the following equations for $0 \le \vartheta \le 2\pi$

(a) $\cos 2\vartheta - \cos\vartheta = -1$ (b) $4\sin 2\vartheta = 5\sin\vartheta$

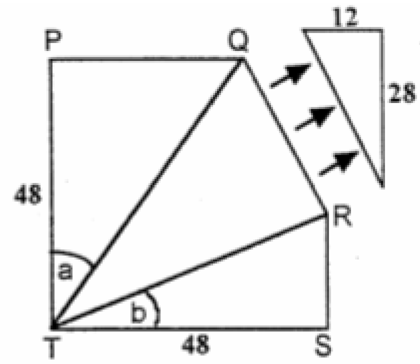
3. Given that $\tan A = \frac{3}{4}$ and $\tan B = \frac{5}{12}$, where (A, $B < \frac{\pi}{2}$), find the exact value of sin(A + B).

4. In the diagram, a square has a triangle cut from one corner. The resulting shape $PQRST$ is a pentagon.

(a) Calculate the lengths of PQ, TQ, RS and TR.

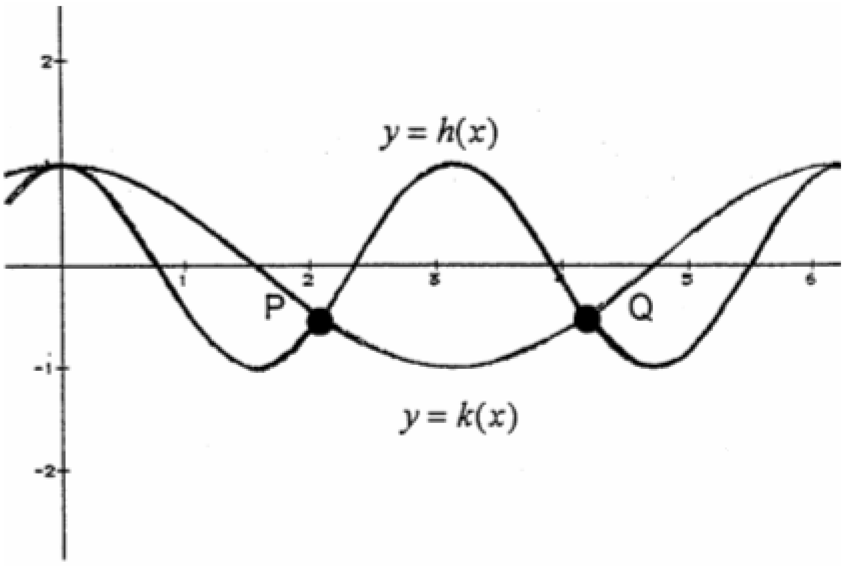
(b) Write down an expression for the size of angle QTR, in terms of a and b.

(c) Show that $\sin QTR = \frac{33}{65}$.



5. (a) Solve the equation $\cos 2x^\circ = \cos x^\circ$ for $0^\circ \le x^\circ \le 360^\circ$.

(b) The diagram below shows parts of the graph of two cosine functions, h and k. State expressions for $h(x)$ and $k(x)$.



(c) Use your answers to part (a) to find the coordinates of P and Q.

(d) Hence state the values of x in the interval $0^\circ \le x^\circ \le 360^\circ$ for which $\cos 2x^\circ < \cos x^\circ$.

COMPOUND ANGLE FORMULAE - SET 2 45 SOLUTIONS

1 a) $\cos 2x^\circ - 3\sin x^\circ = 2\sin^2 x^\circ$

$1 - 2\sin^2 x^\circ - 3\sin x^\circ = 2\sin^2 x^\circ$ 1

$4\sin^2 x^\circ + 3\sin x^\circ - 1 = 0$

$(4\sin x^\circ - 1)(\sin x^\circ + 1) = 0$ 1

$\Rightarrow \quad 4\sin x^\circ - 1 = 0 \quad$ or $\quad \sin x^\circ + 1 = 0$

$\sin x^\circ = \frac{1}{4}$ 1 $\qquad \sin x^\circ = -1$ 1

$x^\circ = 14.5^\circ, 165.5^\circ$ 1 $\qquad x^\circ = 270^\circ$ 1

$x^\circ \in \{14.5^\circ, 165.5^\circ, 270^\circ\}$

b) $3\cos 2x^\circ - 2\sin x^\circ - 1 = 0$

$3(1 - 2\sin^2 x^\circ) - 2\sin x^\circ - 1 = 0$ 1

$3 - 6\sin^2 x^\circ - 2\sin x^\circ - 1 = 0$

$6\sin^2 x^\circ + 2\sin x^\circ - 2 = 0$

$3\sin^2 x^\circ + \sin x^\circ - 1 = 0$ 1

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$$\sin x^\circ = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{(1)^2 - 4(3)(-1)}}{6}$$ 1

$$= \frac{-1 \pm \sqrt{13}}{6}$$

$\sin x^\circ = \frac{-1 + \sqrt{13}}{6}$ 1 or $\sin x^\circ = \frac{-1 - \sqrt{13}}{6}$

$x^\circ = 25.7^\circ, 154.3^\circ$ 1 $\qquad = 230.1^\circ, 309.9^\circ$ 1

$x^\circ \in \{25.7^\circ, 154.3^\circ, 230.1^\circ, 309.9^\circ\}$

2a)

$$\cos 2\theta - \cos\theta = -1$$

$$2\cos^2\theta - 1 - \cos\theta = -1 \qquad 1$$

$$2\cos^2\theta - \cos\theta = 0$$

$$\cos\theta(2\cos\theta - 1) = 0 \qquad 1$$

$\Rightarrow \cos\theta = 0$ 1 or $2\cos\theta - 1 = 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ 1

$2\cos\theta = 1$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$ 1

$$\Rightarrow \theta \in \left\{ \frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \right\} \text{ radians}$$

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b)

$$4\sin 2\theta = 5\sin\theta$$

$$8\sin\theta\cos\theta = 5\sin\theta \qquad 1$$

$$8\sin\theta\cos\theta - 5\sin\theta = 0$$

$$\sin\theta(8\cos\theta - 5) = 0 \qquad 1$$

$\Rightarrow \sin\theta = 0$ 1 or $8\cos\theta - 5 = 0$

$\theta = 0, \pi, 2\pi$ 1

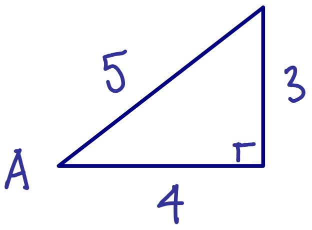
$\cos\theta = 5/8$ 1

$\theta = 51.3 \cdot \frac{\pi}{180}, 308.7 \cdot \frac{\pi}{180}$

$\theta = 0.9, 5.4$ 1

$$\Rightarrow \theta \in \{0, 0.9, \pi, 5.4, 2\pi\} \text{ radians}$$

3. $\tan A = \frac{3}{4}$ $\qquad\qquad$ $\tan B = \frac{5}{12}$



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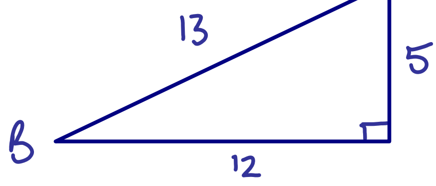

$$\begin{aligned}\sin(A+B) &= \sin A\cos B + \cos A \sin B \\ &= \frac{3}{5}\cdot\frac{12}{13} + \frac{4}{5}\cdot\frac{5}{13} \quad 1 \quad 1 \\ &= \frac{36}{65} + \frac{20}{65} \\ &= \frac{56}{65} \quad 1\end{aligned}$$

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4. a) $PQ = 36$ 1

$TQ = 60$ 1 (by Pythagoras - 12 x 3-4-5 $\triangle$)

$RS = 20$ 1

$TR = 52$ 1 (by Pythagoras - 4 x 5-12-13 $\triangle$)

b) $\angle QTR = 90^\circ - (a+b)^\circ$ 1

c)
$$\begin{aligned}\sin QTR &= \sin(90-(a+b)^\circ) \\ &= \cos(a+b)^\circ \quad 1 \\ &= \cos a^\circ \cos b^\circ - \sin a^\circ \sin b^\circ \\ &= \frac{48}{60}\cdot\frac{48}{52} - \frac{36}{60}\cdot\frac{20}{52} \quad 1 \\ &= \frac{4}{5}\cdot\frac{12}{13} - \frac{3}{5}\cdot\frac{5}{13} \\ &= \frac{48}{65} - \frac{15}{65}\end{aligned}$$

$$\sin QTR = \frac{33}{65} \quad 1$$

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5. a)

$$\cos 2x^\circ = \cos x^\circ$$

$$2\cos^2 x^\circ - 1 = \cos x^\circ \qquad 1$$

$$2\cos^2 x^\circ - \cos x^\circ - 1 = 0$$

$$(2\cos x^\circ + 1)(\cos x^\circ - 1) = 0 \qquad 1$$

$$\Rightarrow \quad 2\cos x^\circ + 1 = 0 \qquad \text{or} \qquad \cos x^\circ - 1 = 0$$

$$\cos x^\circ = -\tfrac{1}{2} \qquad\qquad \cos x^\circ = 1$$

$$x^\circ = 120^\circ, 240^\circ \quad 1 \qquad\qquad x^\circ = 0^\circ, 360^\circ$$

1

$$x^\circ \in \{0^\circ, 120^\circ, 240^\circ, 360^\circ\}$$

b) $h(x) = \cos 2x^\circ$ 1 , $k(x) = \cos x^\circ$ 1

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c) $P(120^\circ, -\tfrac{1}{2})$ $Q(240^\circ, -\tfrac{1}{2})$

1 1

d) $\cos 2x^\circ < \cos x^\circ$ for $0^\circ < x^\circ < 120^\circ$ & $240^\circ < x^\circ < 360^\circ$

1 1